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On nonlinear maps preserving certain spectral quantities of operator products

Abstract :

This thesis is devoted to the study of nonlinear maps on the algebra of bounded linear operators that preserve certain spectral quantities of products of operators. The work is divided into two main parts.

In the first part, we characterize nonlinear maps on $B(X)$, the algebra of bounded linear operators on an infinite-dimensional complex Banach space X , that preserve every spectral function $\Delta(\cdot)$ satisfying

$$\partial(\sigma(\cdot)) \subseteq \Delta(\cdot) \subseteq (\cdot),$$

where $\sigma(\cdot)$ denotes the usual spectrum and $\partial(\sigma(\cdot))$ its boundary. Such a function is referred to as a ∂ -spectrum.

We begin by determining the forms of all surjective maps on $B(X)$, the algebra of bounded linear operators on an infinite-dimensional complex Banach space X , that preserve the ∂ -spectrum of usual product of operators.

When $X = H$ is an infinite-dimensional complex Hilbert space, we further characterize the maps that preserve these spectral functions for the skew-product of operators, while relaxing the surjectivity assumption.

Subsequently, we extend these investigations to generalized products of operators. Specifically, we characterize maps ϕ (not necessarily surjective) on $B(X)$ that satisfy

$$\Delta(T_1 * \dots * T_k) = \Delta(\phi(T_1) * \dots * \phi(T_k)) \text{ for all } T_1, \dots, T_k \in B(X),$$

where Δ is a ∂ -spectrum map.

In the second part of this thesis, we study surjective maps on the algebra of all bounded linear operators that preserve the ascent or descent of certain products of operators.

We first provide a complete description of surjective mappings $\phi : B(H) \rightarrow B(H)$ satisfying

$$\text{asc}(AB^*A) = \text{asc}(\phi(A)\phi(B)^*\phi(A)) \text{ for all } A, B \in B(H),$$

or,

$$\text{desc}(AB^*A) = \text{desc}(\phi(A)\phi(B)^*\phi(A)) \text{ for all } A, B \in B(H),$$

where H is an infinite-dimensional complex Hilbert space.

Finally, we characterize surjective maps $\phi : B(X) \rightarrow B(X)$ that preserve the ascent or descent of the Jordan product, i.e., those satisfying

$$\text{asc}(AB + BA) = \text{asc}(\phi(A)\phi(B) + \phi(B)\phi(A)) \text{ for all } A, B \in B(X),$$

Or,

$$\text{desc}(AB + BA) = \text{desc}(\phi(A)\phi(B) + \phi(B)\phi(A)) \text{ for all } A, B \in B(X).$$

Keywords: Nonlinear preserver problems, Banach algebras, ∂ -spectrum, Rank-one operators,

Product of operators, Skew product, Jordan product, Generalized product, Ascent, Descen

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